On consistency of the closed bosonic string with different left-right ordering constants

A.A. Deriglazov*

Instituto de Física, Universidade Federal do Rio de Janeiro,
Rio de Janeiro, Brasil.

Abstract

Closed bosonic string with different normal ordering constants $a \neq \bar{a}$ for the right and the left moving sectors is considered. One immediate consequence of this choice is absence of tachyon in the physical state spectrum. Selfconsistency of the resulting model in the "old covariant quantization" (OCQ) framework is studyed. The model is manifestly Poincare invariant, it has non trivial massless sector and is ghost free for $D = 26, \ a = 1, \ \bar{a} = 0$. A possibility to obtain the light-cone formulation for the model is also discussed.

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For the case of the open bosonic string, presence of tachyon is necessary condition if one wishes to obtain a theory with a nontrivial massless sector presented. The same conclusion follows from a lot of selfconsistency checks which can be performed by using of various quantization schemes [1-11]. For the closed string there is exist simple possibility to remove tachyon from the physical spectrum. Namely, it is sufficiently to choose different values of constants in the normal ordered expressions for the constraints: $L_0 - a = 0, \ \bar{L}_0 - \bar{a} = 0, \ a \neq \bar{a}$. Then the constraint $L_0 - \bar{L}_0 - (a - \bar{a}) = 0$ prohibites appearance of the tachyon among the physical states.

*alexei@if.ufrj.br On leave of absence from Dept. of Math. Phys., Tomsk Polytechnical University, Tomsk, Russia

The aim of this Letter is to discuss selfconsistency of the resulting model. The model will be considered first in the OCQ framework. The massless sector is presented in the case of any nonnegative integer a, $\bar{a}$. Further restrictions on the constants arise from the no ghost theorem. In addition to the standard choise, one finds the unique possibility $a=1,\ \bar{a}=0$. The ground state of the resulting theory turns out to be massless vector. Proof of the no ghost theorem will be discussed in some details (while the result $a=1, \bar{a}=0$ can be deduced, in principle, from the literature [1-6, 10-12]). It is motivated by the fact that the standard proof implies introduction of a sufficiently complicated machinery. In particular, one needs to consider the discretised (or the admissible [10]) momentum space and DDF states and then to establish relation among the DDF space and the total state space. Below more direct proof which do not involve of these tools will be presented. Also, exact relation among the space-time dimension and the ordering constants is find (see Eq.(25) below). In conclusion I consider also a possibility to remove anomaly in the light cone Poincare algebra for the model under investigation.

In the gauge $g^{ab}=\eta^{ab}=(-,+)$ general solution of the equations of motion for the closed bosonic string is (we use the standard conventions [10]: $x^\mu(\tau,\sigma+\pi)=x^\mu(\tau,\sigma),\ \eta_{\mu\nu}=(-,+,\ldots,+)$)

$$x^\mu(\tau,\sigma)=X^\mu+\frac{1}{\pi T}P^\mu\tau+\frac{i}{2\sqrt{\pi T}}\sum_{n\neq 0}\frac{1}{n}\left[\bar{\alpha}^\mu_n e^{i2n(\tau+\sigma)}+\alpha^\mu_{-n}e^{-i2n(\tau-\sigma)}\right]. \qquad (1)$$

Operators which correspond to the variables $(X,P,\alpha,\bar{\alpha})$ will be denoted by the same symbols. Then the commutation relations are $[X^\mu,\ P^\nu]$ $=-i\eta^{\mu\nu},\quad [\alpha^\mu_n,\ \alpha^\nu_k]=[\bar{\alpha}^\mu_n,\ \bar{\alpha}^\nu_k]=-n\eta^{\mu\nu}\delta_{n+k,0}$, where $\alpha^\mu_0=-\bar{\alpha}^\mu_0\equiv\frac{1}{2\sqrt{\pi T}}P^\mu$. General vector $\mid\Psi>$ of the total Fock space is linear combination of the basic vectors

$$\mid\varphi>=\prod_{m,n=1}^{\infty}\prod_{\mu_m,\nu_n=0}^{D-1}(\alpha^{\mu_m}_{-m})^{s_{m,\mu_m}}(\bar{\alpha}^{\nu_n}_{-n})^{t_{n,\nu_n}}\mid 0,p>, \qquad (2)$$

where $\mid 0,p>\equiv f(p)\mid 0>$ and $P^\mu f(p)=p^\mu f(p)$. Beside that, one has the Virasoro operators which correspond to the first class constraints of the classical theory $(\partial_\tau x^\mu\pm\partial_\sigma x^\mu)^2=0$

$$L_m=\frac{1}{2}\sum_{\forall k}\alpha^\mu_{m-k}\alpha^\mu_k,\quad \bar{L}_m=\frac{1}{2}\sum_{\forall k}\bar{\alpha}^\mu_{m-k}\bar{\alpha}^\mu_k,\quad m\neq 0, \qquad (3)$$

$$L_0 = \frac{1}{8\pi T}P^2 + N, \quad \bar{L}_0 = \frac{1}{8\pi T}P^2 + \bar{N}, \tag{4}$$

where the level number operators are

$$N = \sum_1^\infty \alpha^\mu_{-k}\alpha^\mu_k, \quad \bar{N} = \sum_1^\infty \bar{\alpha}^\mu_{-k}\bar{\alpha}^\mu_k. \tag{5}$$

Transition to the quantum theory is not a unique procedure since the quantities $L_0, \bar{L}_0$ involve products of the non-commuting operators. So one needs in some ordering prescription. An appropriate choice is normal ordering prescription (as it is written in Eq.(5)) which gives well defined operators for the Fock space realization. Then one more step is necessary in the OCQ framework. The normal ordering prescription implies appearance of anomaly terms in the quantum Virasoro algebra, so one needs to use the Gupta-Bleuler quantization prescription by requiring that the physical states are annihilated by half of the operators (3) only

$$L_m \mid ph >= \bar{L}_m \mid ph >= 0, \quad m > 0, \tag{6}$$

as well as by

$$(L_0 - a) \mid ph >= (\bar{L}_0 - \bar{a}) \mid ph >= 0. \tag{7}$$

Here the constants $a, \bar{a}$ (real numbers) correspond to the above mentioned ambiguouty and can be fixed further from the selfconsistency requirements. Since the theory is Poincare invariant, the Fock space generated by (2) is a representation space of the Poincare group. In particular, the operator P^μ is identified with the Poincare shift generator. From this it follows that eugenvalue of the operator $-P^2$ in an irreducible subspace gives mass of the state: $M^2 = -P^2$. Then Eq.(7) can be rewritten in the form

$$M^2 = 4\pi T(N + \bar{N} - (a + \bar{a})), \tag{8}$$

$$N - \bar{N} - (a - \bar{a}) = 0. \tag{9}$$

The standard choice of the constants is, from the beginning: $a = \bar{a}$. Let me take them different. The immediate consequence is that the state

$| 0, p >\equiv f(p) | 0 >$ do not belong to the physical subspace since it do not obeys to Eq.(9).

Our aim now is to discuss properties of this model. Consider conditions which can lead to appearance of the massless sector in the physical spectrum. Eugenvalues of the operators $N, \bar{N}$ in the space (2) are integer non negative numbers, one writes $N | \varphi_0 >= (k+m) | \varphi_0 >,\ \bar{N} | \varphi_0 >=$
$m | \varphi_0 >$. From Eq.(9) it follows $a - \bar{a} = k$, if one wishes to obtain non empty physical sector. From Eq.(8) the mass of the state is $M^2 = 4\pi T\,(k + 2m - (a + \bar{a}))$. Thus the massless sector can be presented if

$$a = k + m, \quad \bar{a} = m; \quad k \geq 0, \quad m \geq 0. \tag{10}$$

An additional restrictions on $a, \bar{a}$ will follow from the no ghost theorem.

It will be convenient to work in the light-cone coordinates: $\alpha^{\mu}_{-m} \equiv (\alpha^{+}_{-m}, \alpha^{-}_{-m}, \alpha^{i}_{-m})$. Since Eq.(2) contains the commuting operators only, they can be arranged in any desired way. On the same reason it is sufficiently to consider only one (left or right) sector. Retaining only one sector, let me fix the following order (and notations)

$$\begin{aligned} | \varphi_R >= \prod_{i=1}^{\infty} (\alpha^{+}_{-i})^{s_i} \left[\prod_{k=1}^{\infty} \prod_{i_k=1}^{D-2} (\alpha^{i_k}_{-k})^{\rho_{k,i_k}} \right] (\alpha^{-}_{-1})^{\lambda_1} \ldots (\alpha^{-}_{-m})^{\lambda_m} \mid 0,\ p > \\ \equiv | (\alpha^{-}_{-1})^{\lambda_1}, \ldots, (\alpha^{-}_{-m})^{\lambda_m} >^{\Lambda} \equiv | \quad >^{\Lambda}, \end{aligned} \tag{11}$$

where $\Lambda = \sum_{r=1}^{m} \lambda_r$ is a number of the operators α^{-}. Then the following statement can be formulated.

<u>Lemma 1.</u> Any basic vector (11) can be presented as a linear combination of the vectors

$$\begin{aligned} | e_R >= L^{\lambda_1}_{-1} \ldots L^{\lambda_m}_{-m} (\alpha^{+}_{-1})^{s_1} \ldots (\alpha^{+}_{-n})^{s_n} \left[\prod_{k=1}^{\infty} \prod_{i_k=1}^{D-2} (\alpha^{i_k}_{-k})^{\rho_{k,i_k}} \right] | 0,\ p > \equiv \\ L^{\lambda_1}_{-1} \ldots L^{\lambda_m}_{-m} K^{s_1}_{-1} \ldots K^{s_n}_{-n} \mid t > . \end{aligned} \tag{12}$$

with some (λ, s, ρ) (in general case they are different from the corresponding factors in Eq.(11). It was denoted $K_{-n} \equiv \alpha^{+}_{-n}$. Also, if $L_0 | \varphi_R >= R | \varphi_R >$, then all the vectors $| e_R >$ are eigenvectors of L_0 with the same eigenvalue

$$L_0 | e_R >= R | e_R >, \quad \forall \quad | e_R > . \tag{13}$$

In other words *all the operators* α^-_{-r} can be incorporated into L_{-r}, which is crucial property for establishing of the no ghost theorem. The proof of Lemma 1 is as follows. Let me start from the "higher order" operators α^-_{-m} which are presented in Eq.(11). One writes the identity[1]

$$\alpha^-_{-m} \mid 0,p>= -\frac{1}{\alpha^+_0} L_{-m} \mid 0,p> + (\alpha^-_{-m} + \frac{1}{\alpha^+_0} L_{-m}) \mid 0,p>, \tag{14}$$

where $\alpha^+_0 = \frac{1}{2\sqrt{\pi T}} p^+ \neq 0$. The bracket on the r.h.s. do not contains α^-_{-m} (note that for $m > 1$ it contains $\alpha^-_{-k},\ k < m$). Note also that it contains in fact a finite number of terms only. By virtue of Eq.(14) the basic vector (11) can be presented as

$$\begin{aligned} \mid\ \ >^\Lambda = c \left[\prod \alpha^+\right] \left[\prod \alpha^i\right] (\alpha^-_{-1})^{\lambda_1} \ldots (\alpha^-_{-m})^{\lambda_m - 1} L_{-m} \mid 0,\ p> + \\ \mid (\alpha^-_{-1})^{\lambda_1'}, \ldots, (\alpha^-_{-m+1})^{\lambda_{m-1}'}, (\alpha^-_{-m})^{\lambda_m - 1} >^\Lambda, \end{aligned} \tag{15}$$

where $c = -\frac{1}{\alpha^+_0}$. Below we will omit unessential numerical factors arising in the process. Now the operator L_{-m} can be moved to the left. Since $\alpha^\mu_{-n} L_{-m} = L_{-m} \alpha^\mu_{-n} - n\alpha^\mu_{-n-m}$, the number Λ in the process can not be changed, and the result is of the form

$$\begin{aligned} \mid (\alpha^-_{-1})^{\lambda_1}, \ldots, (\alpha^-_{-m})^{\lambda_m} >^\Lambda = \\ L_{-m} \mid (\alpha^-_{-1})^{\lambda_1} \ldots (\alpha^-_{-m})^{\lambda_m - 1} >^{\Lambda - 1} + \mid\ \ >^{\Lambda-1} + \\ \mid (\alpha^-_{-1})^{\lambda_1'}, \ldots, (\alpha^-_{-m+1})^{\lambda_{m-1}'}, (\alpha^-_{-m})^{\lambda_m - 1} >^\Lambda, \end{aligned} \tag{16}$$

where all the states $\mid\ \ >$ on the right hand side are linear combinations of the vectors which have the form (11). For the first two terms the number of the operators α^- is decreased by one unit. (note that the second term on the r.h.s. contains in general "higher order" operators $\alpha^-_{-r},\ r > m$). The last term has the same total number of the operators α^- as the initial vector, but *the number of the "higher order" operators* α^-_{-m} *is decreased by one unit.* After numerous repetition of the procedure for the last term one incorporates all the operators $\alpha^-_{-2}, \cdots, \alpha^-_{-m}$ into $L_{-2}, \cdots, L_{-m}$. Then (16) acquires the form

$$\mid\ \ >^\Lambda = \sum_m L_{-m} \mid\ \ >^{\Lambda-1} + \mid\ \ >^{\Lambda-1} + \mid (\alpha^-_{-1})^\Lambda >^\Lambda . \tag{17}$$

[1] Appearance of the factor α^+_0 in the denominator is the standard singularity of the light cone formulation [11].

On the next step one notes that in the identity

$$\alpha^-_{-1} = -\frac{1}{\alpha^+_0} L_{-1} + (\alpha^-_{-1} + \frac{1}{\alpha^+_0} L_{-1}), \tag{18}$$

the bracket on the r.h.s. do not contains the operators α^-_{-m} at all. So on this stage number of the operators α^- in the last term of Eq.(17) begin to decrease, one finds

$$\begin{aligned} \mid (\alpha^-_{-1})^\Lambda >^\Lambda = L_{-1} \mid (\alpha^-_{-1})^{\Lambda-1} >^{\Lambda-1} + \mid (\alpha^-_{-1})^{\Lambda-1} >^{\Lambda-1} = \ldots = \\ L_{-1} \mid (\alpha^-_{-1})^{\Lambda-1} >^{\Lambda-1} + \mid \quad >^0 = \ldots = L^\Lambda_{-1} \mid \quad >^0 + \mid \quad >^0 . \end{aligned} \tag{19}$$

Then Eq.(17) acquires the form

$$\mid \quad >^\Lambda = \sum_m L_{-m} \mid \quad >^{\Lambda-1} + \mid \quad >^{\Lambda-1} + \mid \quad >^0, \tag{20}$$

where all the vectors of the form $L^\Lambda_{-1} \mid \quad >^0$ were included into $\sum L_{-m} \mid \quad >^{\Lambda-1}$. In the result all the vectors $\mid >$ on the r.h.s. of Eq.(20) are of the form (2), but contain less then Λ operators.

Now all the previous procedure can be numeruosly repeated for the vectors $\mid \quad >^{\Lambda-1}$ from (20), with the final result being of the form (12). One notices also that the total level number R can not be changed in the process described, so Eq.(13) holds.

Let me introduce subspaces of (12) as follows

$$K_R = \{\mid k >= K^{s_1}_{-1} \ldots K^{s_n}_{-n} \mid t >\}, \tag{21}$$

$$S_R = \left\{\mid s >= L^{\lambda_1}_{-1} \ldots L^{\lambda_m}_{-m} K^{s_1}_{-1} \ldots K^{s_n}_{-n} \mid t >, \quad \sum_{r=1}^{m} r\lambda_r > 0\right\}. \tag{22}$$

By construction S_R consist of the spurious vectors. Evidently, the space (12) is a direct sum of (21),(22). Further, the transverse vectors $|t>$ have non negative norm and obey the property $K_n|t>= 0, \quad n > 0$, from which it follows

<u>Lemma 2.</u> Vectors of the subspace (21) have nonnegative norm. In particular, $< k \mid k >= 0$ if $\sum_{r=1}^n r s_r > 0$.

Below we use also the following result.

Lemma 3. Vectors of the subspace (21) are linearly independent from the vectors of (22).
Actually, suppose an opposite

$$\mid s >= L_{-1}^{\lambda_1}\ldots L_{-m}^{\lambda_m}K_{-1}^{s_1}\ldots K_{-n}^{s_n}\mid t >= \sum c(\{l\},t^{'})K_{-1}^{l_1}\cdots K_{-k}^{l_k}\mid t^{'} > . \qquad (23)$$

where $\sum_{r=1}^{m} r\lambda_r \equiv s > 0$ and consider action of the operator K_s on both sides of (23). Then the r.h.s. is zero, while the l.h.s. can be computed directly, one finds

$$K_s \mid s >\equiv c\frac{p^+}{2\sqrt{\pi T}}K_{-1}^{s_1}\ldots K_{-n}^{s_n}\mid t >= 0, \qquad (24)$$

where $c > 0$. From this it follows $\mid s >= 0$.
These results allows one to prove the no ghost theorem.
Theorem. Vectors of the physical subspace (6), (7) have nonnegative norm for the case

$$a \leq 1, \quad \bar{a} \leq 1, \quad D \leq \frac{2(2-a)(21-8a)}{3-2a}, \quad D \leq \frac{2(2-\bar{a})(21-8\bar{a})}{3-2\bar{a}}. \qquad (25)$$

The proof is as follows. The physical state can be presented as combination of the basic vectors (2): $\mid ph >= \sum c_i \mid \varphi_i >$. By using of linear independence of the vectors $\mid \varphi_i >$ one concludes that all of them obey to Eq.(7) with the same eugenvalue as the vector $\mid ph >$. In its turn, $\mid \varphi_i >$ can be presented as combination of the vectors $\mid e_R > \times \mid e_L >$ according to Lemma 1. All of them obey also to Eq.(7). Thus one writes $\mid ph >=\mid k > + \mid s >, \mid k >\subset K \equiv K_R \times K_L, \mid s >\subset S \equiv S_R \times S_L$, where $\mid k >, \ \mid s >$ obey to Eq.(7). Further, the operator $L_{-m}(\bar{L}_{-m})$ with $m > 2$ can be presented through $L_{-1}, L_{-2}, (\bar{L}_{-1}, \bar{L}_{-2})$ or, equivalently, through [2] $L_{-1}, \tilde{L}_{-2}$ $(\bar{L}_{-1}, \tilde{\bar{L}}_{-2})$, where

$$\tilde{L}_{\pm 2} = L_{\pm 2} + cL_{\pm 1}^2, \quad c = \frac{3}{2(3-2a)};$$
$$\tilde{\bar{L}}_{\pm 2} = \bar{L}_{\pm 2} + \bar{c}\bar{L}_{\pm 1}^2, \quad \bar{c} = \frac{3}{2(3-2\bar{a})}. \qquad (26)$$

It allows one to rewrite the physical state as

$$\mid ph >=\mid k > + L_{-1}\mid \chi_1 > + \tilde{L}_{-2}\mid \chi_2 > + \bar{L}_{-1}\mid \bar{\chi}_1 > + \tilde{\bar{L}}_{-2}\mid \bar{\chi}_2 > . \qquad (27)$$

[2] Application of the operators $\tilde{L}_{\pm 2}$ instead of $L_{\pm 2}$ is the standard technical moment. The operator $\tilde{L}_2$ is chosen in such a way that $[\tilde{L}_2, \ L_{-1}]\mid \chi_1 >= 0$, which simplifies the proof below.

The norm of the state is

$$< ph \mid ph >=< k \mid k > + < \chi_1 \mid L_1 | k > + < \chi_2 \mid \tilde{L}_2 \mid k > + \\ < \bar{\chi}_1 \mid \bar{L}_1 | k > + < \bar{\chi}_2 \mid \tilde{\bar{L}}_2 \mid k > . \tag{28}$$

Thus one needs to know $L_i|k>$. This information can be obtained from the condition that $|ph>$ is the physical vector. Namely, one finds after some algebra

$$L_1 \mid ph >= 0 \implies \mid k_1 > -2(1-a) \mid \chi_1 > + \mid s_1 >= 0, \tag{29}$$

$$\tilde{L}_2 \mid ph >= 0 \implies \mid k_2 > -A \mid \chi_2 > + \mid s_2 >= 0, \tag{30}$$

$$\bar{L}_1 \mid ph >= 0 \implies \mid \bar{k}_1 > -2(1-\bar{a}) \mid \bar{\chi}_1 > + \mid \bar{s}_1 >= 0, \tag{31}$$

$$\tilde{\bar{L}}_2 \mid ph >= 0 \implies \mid \bar{k}_2 > -\bar{A} \mid \bar{\chi}_2 > + \mid \bar{s}_2 >= 0, \tag{32}$$

where

$$\mid k_1 >= L_1 \mid k >, \quad \mid k_2 >= \tilde{L}_2 \mid k >, \quad \mid k_i > \subset K; \\ \mid \bar{k}_1 >= \bar{L}_1 \mid k >, \quad \mid \bar{k}_2 >= \tilde{\bar{L}}_2 \mid k >, \quad \mid \bar{k}_i > \subset K; \\ \mid s_i > \subset S, \quad \mid \bar{s}_i > \subset S. \tag{33}$$

It was also denoted

$$A(c,a) = 4(c^2+3c+1)(2-a) - 8c^2(2-a)^2 - \frac{1}{2}D, \quad \bar{A} \equiv A(\bar{c},\bar{a}). \tag{34}$$

It remains to consider various possibilities for the vectors $\mid \chi_i >, \mid \bar{\chi}_i >$. Let $\mid \chi_1 >$ do not contains of the operators $L_{-m}, \bar{L}_{-n}$: $\mid \chi_1 >=\mid k_0 > \subset K$, while $\mid \chi_2 >, \mid \bar{\chi}_1 >, \mid \bar{\chi}_2 > \subset S$. From Lemma 3 and Eqs.(29)-(33) it follows

$$L_1 \mid k >= 2(1-a) \mid k_0 >, \quad \mid s_1 >= 0, \\ \tilde{L}_2 \mid k >= \bar{L}_1 \mid k >= \tilde{\bar{L}}_2 \mid k >= 0, \\ \tilde{L}_2 \mid s >= \bar{L}_1 \mid s >= \tilde{\bar{L}}_2 \mid s >= 0. \tag{35}$$

While $\mid s >$ is not a physical state, Eq.(35) allows one to estimate the norm (28), namely

$$< ph \mid ph >=< k \mid k > +2(1-a) < k_0 \mid k_0 > . \tag{36}$$

According to Lemma 2, the condition $a \leq 1$ garantees that $< ph \mid ph >\geq 0$.

Other possibilities can be analysed in a similar way. The result is that the conditions $1-a \geq 0$, $1-\bar{a} \geq 0$, $A \geq 0$, $\bar{A} \geq 0$ garantee non negative norm of the physical state. From this it follows Eq.(25).

Let me return to the discussion of the model (1),(10). Besides the standard choice $a = \bar{a} = 1$, the only other possibility which is consistent with the no-ghost theorem is [3] $a = 1$, $\bar{a} = 0$, which implies also $D \leq 26$ (see Eqs.(10), (25)). The requirement of absence of the Weyl anomaly implies $D = 26$. Now the lowest mass sector is the massless one and is selected by

$$(N - \bar{N} - 1) \mid \varphi_0 >= 0, \quad (N + \bar{N} - 1) \mid \varphi_0 >= 0, \tag{37}$$

as well as by Eq.(6). The standard reasoning [12] shows that the ground state is the equivalence class

$$\mid \widetilde{\varphi_0} > = \left\{ [f_\mu(p) + d(p)p_\mu]\alpha^\mu_{-1} \mid 0 >, \quad p^2 = 0, \quad f_\mu p^\mu = 0, \quad \forall d(p) \right\}, \tag{38}$$

which corresponds to the massless vector particle with $D-2$ transverse physical polarisations. In the result the closed string with $a = 1$, $\bar{a} = 0$ has the same massless sector as the open string. The second, third, ... massive levels consist of states of the form $(\alpha_{-1})^2\bar{\alpha}_{-1} \mid 0 >$, $(\alpha_{-1})^3(\bar{\alpha}_{-1})^2 \mid 0 >, \ldots$.

Thus, in the OCQ framework the closed bosonic string for the case $a = 1, \bar{a} = 0$, $D = 26$ is the ghost free manifestly Poincare invariant theory without tachyon and with massless vector particle being its ground state. Note also that it seems to be consistent at least on the level of tree amplitudes since the corresponding calculations can be performed in the OCQ framework [9].

In conclusion, consider a possibility of the light cone formulation for the model. In this case one impose the gauge $\alpha^+_m = \bar{\alpha}^+_m = 0$, $m \neq 0$, $X^+ = 0$ for the constraints $L_m = \bar{L}_m = L_0 + \bar{L}_0 = 0$ of the classical theory. Then the remaining physical degrees of freedom are $X^-, P^+, X^i, P^i, \alpha^i_m, \bar{\alpha}^i_m$. The Dirac bracket for these variables coincides with the Poisson one. The remaining constraint involve

[3] An equivalent choice is, of course, $a = 0$, $\bar{a} = 1$.

the transverse oscillators only [11]

$$L_{0,tr} - \bar{L}_{0,tr} - \bar{b} = 0. \tag{39}$$

Here some ordering constant $\bar{b}$ was included. In this formulation the ghosts are absent by construction, but one needs to check Poincare invariance of the resulting quantum theory. The only dangerous commutator of the Poincare algebra is known to be $[J^{i-}, J^{j-}]$, which must be zero. Manifest form of the generators are

$$J^{i-} = \frac{2\pi T}{p^+}(L_{0,tr} + \bar{L}_{0,tr} - b) - X^- P^i + \frac{2i\sqrt{\pi T}}{p^+}(S^{i-} - \bar{S}^{i-}), \tag{40}$$

where $S^{i-} = \sum_{n=1}^{\infty} \frac{1}{n}(\alpha^i_{-n} L_{n,tr} - L_{-n,tr}\alpha^i_n)$, and b is the second ordering constant of the light cone formulation. For the case $D = 26$ the commutator can be presented as

$$[J^{i-},\ J^{j-}] = \frac{4\pi T}{(p^+)^2}\left\{(L_{0,tr} - \bar{L}_{0,tr} + b)S^{ij} - (L_{0,tr} - \bar{L}_{0,tr} - b)\bar{S}^{ij} - 2S^{ij} - 2\bar{S}^{ij}\right\}, \tag{41}$$

where the last two terms is the anomaly resulting from reordering of operators in the process. The standard choice is $b = 2$, $\bar{b} = 0$, which gives the desired result $[J^{i-}, J^{j-}] = 0$ as a consequence of Eq.(39). Other formal possibility is as follows: consider the normal ordering prescription for the operators α^i_m and the Weyl prescription for $\bar{\alpha}^i_m$. Then the anomaly terms in the left sector are absent, and the commutator is proportional to $(L^N_{0,tr} - \bar{L}^W_{0,tr} + b)S^{ij} - (L^N_{0,tr} - \bar{L}^W_{0,tr} - b)\bar{S}^{ij} - 2S^{ij}$. It will be zero if one takes $b = \bar{b} = 1$.

It is known that the Weyl quantization (of the both sectors) for the ordinary string is not consistent (this fact can be extracted also from Eq.(41)). The problem of the Weyl quantization was investigated also in context of the null string [13-16]. From the previous discussion it follows that namely the Weyl quantization can be interesting for the light cone formulation of the model under investigation. This problem will be considered in a separate publication.

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